Series Strategy

In general, you will decide what convergence test to apply based on the form of the series. These are some good general guidelines to get started, but they are not an all-encompassing resource. Use your best judgment.

- 1. If it looks like $\lim_{n \to \infty} a_n \neq 0$, use the Divergence Test.
- 2. If the series is a *p*-series or geometric series, $\sum_{n=1}^{\infty} \frac{1}{n^p}$ or $\sum_{n=0}^{\infty} ar^n$, apply the rules for convergence of *p*-series or geometric series. You may need to do some algebra to get the series into the appropriate form.
- 3. If the series appears to be close to either of these (fractions of sums or differences of terms like n^p or r^n), try a comparison test or limit comparison test. Look at the leading order terms to find a series to compare to.
- 4. If the series is alternating, e.g. $\sum_{n=1}^{\infty} (-1)^n b_n$ or $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$, try the Alternating Series Test.
- 5. If the series involves factorials or other products, try the Ratio Test. Remember that the ratio test is always inconclusive for rational functions of n.
- 6. If $a_n = (b_n)^n$, try the root test. (More generally, the root test is good if your expression involves n^{th} powers.)
- 7. If your series has both positive and negative terms but is not alternating, try testing $\sum_{n=1}^{\infty} |a_n|$ for convergence using an appropriate method, often a comparison test (checking for absolute convergence).
- 8. If $a_n = f(n)$ and you can evaluate $\int f(x) dx$, try the Integral Test.